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Introduction to ''Approximate Riemann Solvers, Parameter Vectors, and Difference Schemes''

The publication of this paper was a watershed event in the practice of computational fluid dynamics (CFD) over the past 20 years, for both the field and the author. Phil Roe, a former Chief Editor of the *Journal of Computational Physics,* is now considered one of the most eminent practitioners of computational fluid dynamics. However, in the 1970s the author was an unknown theoretical aerodynamicist working at the Royal Aircraft Establishment Bedford, U.K., a subsidiary of RAE Farnborough. His interest in the numerical aspects of CFD drew him to ask fundamental questions about the solution of the Euler equations, the well-known system of nonlinear equations governing inviscid compressible aerodynamics. The question of bridging the gulf between the Euler equation system and the scalar nonlinear wave equation within a numerical context while maintaining the desirable shock capturing properties of conservative schemes eventually led him to the ideas contained in the classic paper reprinted here. The author showed that within a simple finite volume context the Euler equations could be locally linearized and still keep the conservation property for the discretized equations which is crucial in obtaining the correct location of shocks. Moreover, the Roe linearization gives the exact solution to the Riemann problem in the case where it consists of a single shock. In his paper the author demonstrated that for the Euler equations much of the work involved was simply a matter of preprocessing and gave full details of the algebra necessary to carry it out. The resulting linearized equations gave the first appearance of the now well-known and widely used ''Roe matrix.''

His familiarity with the work of Godunov made him realize that his discrete decomposition was in effect an approximate Riemann solver, to be contrasted with Godunov's exact solver. As many *Journal of Computational Physics* readers now appreciate, approximate Riemann solvers are extremely useful building blocks in the numerical solution of the Euler equations and other sets of conservation laws.

In this paper, the new approximate Riemann solver was applied to Sod's shock tube problem with discontinuous constant states. In tandem with the construction of the approximate Riemann solver, the author had developed upwind finite volume methods for the scalar nonlinear wave equation with switches at changes in sign of the wavespeed. When these techniques were used in combination and applied to the Sod problem, he obtained remarkable results which were the best available at the time and which set the standard for others to follow.

While recommending the conservative variables as the natural set of variables for the Euler equations, the author was clearly intrigued (as many had been before him) by the special algebraic structure of the Euler flux function. During his investigations he had discovered a set of variables (the parameter vector) in terms of which the flux function is quadratic, and these variables characterized the steps of the algebra needed to define the Roe matrix in the paper. Although there are other ways of generating the preprocessing steps, the emergence of the parameter vector has remained significant and has played an important part in recent years in the generalization to higher dimensions.

To the present day the balance of analytic and numerical properties in this work, and in particular the fluctuation-signal philosophy implied in the paper, has continued to influence code developers. Applications of this technique to a variety of other hyperbolic systems have been developed. Roe solvers for shallow water equations, multiphase flow, nonlinear elasticity, relativistic hydrodynamics, and magnetohydrodynamics are all available and are considered standard tools of the trade. Although the Roe solver is a technique for approximately solving a one-dimensional Riemann problem, this is also a crucial step in many numerical methods for multidimensional problems, and these techniques remain indispensible. The publication of this paper released an awareness of the power of this strategy and eventually led to a flood of papers and applications.

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